Thermal conductivity in relation to porosity and hardness of terrestrial porous media

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Accepted 16 November 2007
Available online 8 January 2008

Abstract

Regression equations for predicting thermal conductivity based on easily measured or available porosity, penetration resistance and content of sand with respect to variously textured terrestrial soils, and porosity and hardness with respect to snow are presented. It is shown that with respect to single soils the performance of regression equations based on penetration resistance and porosity was quite satisfactory ($R^2 = 0.923–0.968$, RMSE = 0.067–0.138 W m$^{-1}$ K$^{-1}$) and was improved after adding a content of sand ($R^2 = 0.958–0.968$, RMSE = 0.055–0.102 W m$^{-1}$ K$^{-1}$). When the data of all the soils were analysed together, the performance of the regression equations based on penetration resistance and porosity was unsatisfactory ($R^2 = 0.399$) and was substantially improved after adding the content of sand ($R^2 = 0.946$). The performance of the regression equations for snow thermal conductivity based on the index of hardness and porosity was better ($R^2 = 0.657$) than that based only on density ($R^2 = 0.501$) [Sturm, M., Perovich, D.K., Holmgren, J., 2002. Thermal conductivity and heat transfer through the snow on the ice of the Beaufort Sea. J. Geophys. Res. 107 (C21), 8043]. The suggested regression equations can help in minimising uncertainties associated with measurements in extraterrestrial media.

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Keywords: Thermal conductivity; Porosity; Hardness; Soil; Snow; Regression equations

1. Introduction

Porous media like mineral aggregates, porous water ice or carbon dioxide ice are common constituents of planets and comets in the solar system (Gori and Corasaniti, 2004). The thermal conductivity of the media is an important factor influencing the energy balance of planetary near surface layers and helps to understand the character of thermal evolution of planetary objects and comet nuclei (Kömle et al., 1991; Sirono and Yamamoto, 1997).

Direct measurements of the thermal conductivity using “heated thin wire” (Bristow et al., 2001) and similar linear heat source methods (Seiferlin et al., 1996; Marczewski et al., 2004) seem most suitable to be used on a planetary surface. However, the use of the methods is limited with respect to porous and low conducting regolith layers due to the smaller conductivity of the materials out of the sensor itself and insufficient contact between the sensor and the surrounding medium, and to overcome the constraints the research is being conducted to optimise the performance of the thermal sensors (Kömle et al., 2006).

Another alternative approach can be the development of regression equations relating the thermal conductivity with more easily and quickly measured or available quantities. A significant parameter influencing the thermal conductivity is the size of the contacts between neighbouring grains (Seiferlin et al., 2003). The contacts are related to penetration resistance and porosity. Some results indicate that the measurements of penetration resistance reflect bonding between the grains of both mineral porous media (Whalley et al., 2000; Lipiec and Hatano, 2003) and snow (Sturm et al., 2002). Moreover, penetrometry data can be useful to reconstruct the grain size distribution (Kargl and Zöhrer, 2006) and aggregate size distribution (Pawlowski et al., 1996) of porous materials by applying mathematical and statistical methods. The significant effect of water...
content on the thermal conductivity has been reported with respect to terrestrial soils (Kersten, 1949; de Vries, 1963). However, in some studies the thermal conductivity was more strongly correlated with air-filled porosity than water content (Ochsner et al., 2001; Usowicz et al., 2006a) and porosity with respect to snow (Sturm et al., 2002) and porous ice under space conditions (Seiferlin et al., 1996). In a study by Ochsner et al. (2001), this effect was linked to the extremely low thermal conductivity of air compared to other components of the porous media. More recently, Seiferlin et al. (2006) reported that heat transport by gas and vapour in the comets and Martian soils can be largely influenced by microstructure and pore size. The pore size is of particular importance when the pores are smaller than the mean free path of gas molecules. The data on porosity are helpful in the interpretation of the weathering of planetary objects and formation and of the evolution of comets, planetoids and meteorites (Strait and Consolmagno, 2005).

Recent attempts undertaken to relate the thermal conductivity of terrestrial soils with penetration resistance and porosity revealed statistically significant relationship \( R^2 = 0.942 \) between the variables (Usowicz et al., 2006a). The authors indicated that the accuracy of the regression equation could be further improved by adding a content of sand.

The foregoing supports developments of combined measurement systems of penetration resistance, thermal conductivity and other quantities influencing the thermal properties of porous media. An example can be the MUPUS thermal and mechanical properties probe (Spohn et al., 2007). Combined measurement approaches provide limited invasive action and are most suitable in space missions where volume and mass of the equipment are confined.

Considering that the extraterrestrial objects do not contain a liquid phase but the main thermal bridges are formed by contacts between the grains and that penetration resistance reflects the contacts and size of the grains (Sturm et al., 2002; Lipiec and Hatano, 2003; Kargl and Zöhrer, 2006), it seems that relating thermal conductivity with the resistance, porosity and the main (textural) component of the porous media is reasonable.

In this paper, we shall present the relationships between the thermal conductivity and easily measured penetration resistance (or hardness) and porosity in variously grained terrestrial soils and snow.

### 2. Materials and methods

#### 2.1. Data sets

The study was based on data from three terrestrial soils and snow. The soils used come from the following sites: Sobieszyn sandy loam (51°35′N 22°09′E, 100 m a.s.l.), Felin silt loam (A) (51°13′N 22°37′E, 190 m a.s.l.), both situated on flat areas in S–E Poland, and Piedmont silt loam (B) (45°3’42”N 7°57’5”E (450 m a.s.l.) in N–W Italy on an 18% slope (Ferrero et al., 2005). The Felin silt loam (A) compared to the Piedmont silt loam (B) is characterised by a greater content of sand (Table 1). Mean air temperatures and yearly precipitation both in Sobieszyn and Felin were 6.1 °C and 472.5 mm, respectively, and those in Piedmont 11.3 °C and 840 mm. Soil temperature at sampling ranged from 10 to 25 °C.

The data on soil hardness, as characterised by penetration resistance, water content, bulk density and sand (2–0.02 mm), were used as independent variables in the regression equations. They were measured in the sandy loam and silt loam (A) soils, and for the silt loam (B) were taken from the literature (Ferrero et al., 2005). The penetration resistance of all three soils was measured by means of a recording penetrometer with a conical probe of 30° angle at penetration rate 6 mm s\(^{-1}\) to a depth of 25 cm. The ranges of the values of the properties for individual soils are given in Table 1.

As to snow, the values of the measured index of hardness, bulk density (Table 1) and thermal conductivity were taken from the literature (Sturm et al., 2002) and were used to develop regression equation for predicting the thermal conductivity. The authors used a heated needle probe to measure the thermal conductivity and porosity of

### Table 1

Ranges of the properties of the terrestrial soils and snow investigated

<table>
<thead>
<tr>
<th>Location</th>
<th>PR (kPa)</th>
<th>( \theta ) ( (m^3 m^{-3}) )</th>
<th>( \rho ) ( (10^3 kg m^{-3}) )</th>
<th>( \rho_s ) ( (10^3 kg m^{-3}) )</th>
<th>Texture ( (10^3 kg kg^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy loam</td>
<td>30–7630</td>
<td>0.046–0.253</td>
<td>1.321–1.824</td>
<td>2.63–2.64</td>
<td>880–900</td>
</tr>
<tr>
<td>Silt loam (A)</td>
<td>43–5020</td>
<td>0.056–0.299</td>
<td>1.127–1.741</td>
<td>2.61–2.63</td>
<td>680</td>
</tr>
<tr>
<td>Silt loam (B)</td>
<td>100–9300</td>
<td>0.096–0.474</td>
<td>0.96–1.48</td>
<td>2.43–2.58</td>
<td>299–347</td>
</tr>
</tbody>
</table>

**Index of hardness**

<table>
<thead>
<tr>
<th>Snow(^a)</th>
<th>1–6</th>
<th>–</th>
<th>0.145–0.515</th>
<th>0.917</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow(^b)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

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PR—penetration resistance, \( \theta \)—water content, \( \rho \)—bulk density and \( \rho_s \)—particle density of soil or ice. Textural components: sand (2–0.02 mm), silt (0.02–0.002 mm), clay (<0.002 mm) and OM—organic matter.

\(^a\)Data from Ferrero et al. (2005).

\(^b\)Data from Sturm et al. (2002).
snow, calculated on the basis of snow density and ice density \((0.917 \times 10^3 \text{ kg m}^{-3})\).

As can be seen from Table 1, the data represent a wide range of values and thereby give surety that the obtained regression equations will allow predicting the thermal conductivity of porous media under various environmental conditions.

Since measurements of the thermal conductivity in situ are difficult, they were done for part of the soil samples under laboratory conditions at different bulk density and water content. These data of bulk density and water content as well as those of remaining samples were used to estimate the thermal conductivity using the statistical-physical model (Usowicz, 1992). It was shown that predictions of the thermal conductivity by means of the model, for a wide range of terrestrial soil types and at various water content, bulk density and temperature \((T)\) were in a good agreement with the above measured values in laboratory \((R^2\) from 0.948 to 0.987; RMSE (root mean square error) from 0.083 to 0.132 W m\(^{-1}\) K\(^{-1}\) (Usowicz et al., 2006b) and with those of the widely used De Vries model (De Vries, 1963). Taking into consideration the good performance of the statistical-physical model, we used its output thermal conductivities as reference \((\text{dependent variable})\) for the conductivities derived from regression equations developed in the present study. In subsequent parts of the paper the reference data will be called “observed data”. Because measurements of penetration resistance and water content \((\text{e.g. by Time Domain Reflectometry})\) compared to bulk density are easier and faster we attempted to obtain reliable regression equations relating thermal conductivity with penetration and porosity in porous media.

### 2.2. Regression equations

Earlier studies showed that the accuracy of the thermal conductivity predictions based on independent variables such as penetration resistance, water content and density of porous media is not satisfactory and can be considerably improved by replacing water content by porosity (Ochsner et al., 2001; Usowicz et al., 2006a).

In this work, two regression Eqs. (1) and (2) providing the most accurate predictions for the thermal conductivity of soil were suggested. To predict the thermal conductivity of snow, Eq. (3) was proposed and compared with Eq. (4) developed by Sturm et al. (2002). Parameters of the proposed equations in this study \((\text{Eqs. (1)–(3)})\) were derived using the Fixed Non-linear Regression procedure in the program Statistica 6 (StatSoft, Inc., 2004). The use of this procedure was supported by earlier results of Côté and Konrad (2005) and Usowicz et al. (2006a) indicating non-linear relation between the thermal conductivity and some physical quantities of porous media. The equations are based on penetration resistance and porosity \((\text{defined as the difference of total porosity and volumetric water content})\). Moreover, the content of sand was included because of its significant effect on the thermal conductivity and spatial variability in the terrestrial fields. The equations are:

\[
\lambda = a \times \sqrt{PR} + b \times (\phi - \theta) + c \quad (1)
\]

\[
\lambda = a \times \sqrt{PR} + b \times (\phi - \theta) + b_1 Q + c \quad (2)
\]

\[
\lambda = a \times \exp(H_c) + b \times \phi + c \quad (3)
\]

\[
\lambda = 0.138 - 1.01 \rho + 3.233 \rho^2 \quad (4)
\]

where \(\lambda\) \((\text{W m}^{-1}\text{ K}^{-1})\) is the thermal conductivity;\( a, b, b_1, c \) are the parameters that are denoted as B coefficients; \(PR\) \((\text{kPa})\) is the penetration resistance; \(\theta\) \((\text{m}^3 \text{ m}^{-3})\) is the volumetric soil water content; \(H_c\) index of hardness (Sturm et al., 2002); \(\phi = 1(\rho/\rho_s)\) \((\text{m}^3 \text{ m}^{-3})\) is the total soil or snow porosity \((\rho)\) is the bulk density of soil or snow and \(\rho_s\) is the soil particle density or ice density, one can accept that \(\phi = \theta_s\) where \(\theta_s\) \((\text{m}^3 \text{ m}^{-3})\) is the soil water content at saturation, in the case of snow the porosity corresponds to total porosity since \(\theta\) equals zero) and \(Q\) \((\text{m}^3 \text{ m}^{-3})\) is the sand fraction \((2-0.02 \text{ mm})\) content in a volume unit of soil that was calculated based on the following equation:

\[
Q = \frac{V_q}{V_c} = \frac{\rho V_q}{\rho_s V_s} = \frac{\rho x_q}{\rho_q} \quad (5)
\]

where \(V_q\) \((\text{m}^3 \text{ m}^{-3})\) is the volumetric content of sand accepted as “quartz”; \(V_c\) \((\text{m}^3 \text{ m}^{-3})\) is the volumetric content of soil, \(V_s\) \((\text{m}^3 \text{ m}^{-3})\) is the volumetric content of solid phase, \(\rho_q\) is the quartz density \((2650 \text{ kg m}^{-3})\) and \(x_q\) \((\text{kg kg}^{-1})\) is the weight content of the sand consisted mostly of quartz.

Statistical evaluation of the agreement between observed and predicted thermal conductivities was performed using the RMSE, the maximum relative error \((\text{MRE})\) and the determination coefficients \((R^2)\) (Usowicz et al., 2006a).

### 3. Results and discussion

Our earlier study showed that porosity and penetration resistance compared to water content and penetration resistance were better predictors of the thermal conductivity of the single terrestrial soil using the regression equations (Usowicz et al., 2006a). The predictions were further better after the sand content was added. For this reason, we used in this study, the regression equations based on penetration resistance, porosity and content of sand for various soils. Parameters of the resulting equations and statistical parameters describing the performance of the regression equations for mineral soils and snow are given in Tables 2 and 3. Eqs. (2) and (3) were selected for graphical presentation and more detailed interpretation because they predicted well the thermal conductivity. Eq. (1) was characterized by statistical parameters (Table 3).

Values of beta coefficients (Table 2) demonstrate that the relative contribution of porosity, \(\phi\), \((-0.644 \text{ to } -0.991)\) in the prediction of the thermal conductivity is substantially greater than that of penetration resistance, \(PR\),
After adding the content of sand, $Q$, to the regression equations, the maximum values of $\beta$ still remained for porosity ($0.009 \text{ to } 0.009 \text{ to } 0.951$) whereas those for the content of sand ($0.048 \text{ to } 0.749$) were greater than for the penetration resistance ($0.057 \text{ to } 0.034$).

In each soil, the values of $\beta$ for porosity and penetration resistance were greater in the equations without than with sand (Table 2). In the sandy loam and silt loam (A) soils, with greater content of sand ($0.537 \text{ m}^3 \text{ m}^{-3}$ and $0.390 \text{ m}^3 \text{ m}^{-3}$, respectively), the sign of $\beta$ for the penetration resistance was negative, in contrast to the silt loam (B) soil of lower content of sand ($0.152 \text{ m}^3 \text{ m}^{-3}$). The negative sign of $\beta$ indicates negative relationship between the thermal conductivity and penetration resistance. Decreasing values of $\beta$ for penetration resistance in sandy loam and silt loam (A) soils with increasing content of sand can be a result of differentiated effect of soil water content and bulk density on penetration resistance depending on the content of sand decreasing soil cohesion. The lower and negative values of $\beta$ for

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Sandy loam</th>
<th>Silt loam (A)</th>
<th>Silt loam (B)</th>
<th>Sandy loam + silt loam (A)</th>
<th>Sandy loam + silt loam (A) + silt loam (B)</th>
<th>Snow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (1)</td>
<td>Beta</td>
<td>B</td>
<td>Beta</td>
<td>B</td>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>3.877</td>
<td>2.794</td>
<td>1.471</td>
<td>2.797</td>
<td>1.701</td>
<td></td>
</tr>
<tr>
<td>$PR$ (kPa)</td>
<td>0.086</td>
<td>0.003</td>
<td>0.009$^a$</td>
<td>0.000$^a$</td>
<td>0.100</td>
<td></td>
</tr>
<tr>
<td>$\phi-\theta$ (m$^3$ m$^{-3}$)</td>
<td>-0.943</td>
<td>-7.646</td>
<td>-0.983</td>
<td>-4.967</td>
<td>-0.991</td>
<td></td>
</tr>
<tr>
<td>$Q$ (m$^3$ m$^{-3}$)</td>
<td>0.295</td>
<td>4.167</td>
<td>0.048$^a$</td>
<td>0.797$^a$</td>
<td>0.178</td>
<td></td>
</tr>
<tr>
<td>Eq. (2)</td>
<td>Intercept</td>
<td>1.542</td>
<td>2.488</td>
<td>0.988</td>
<td>1.431</td>
<td></td>
</tr>
<tr>
<td>$PR$ (kPa)</td>
<td>-0.055</td>
<td>-0.002</td>
<td>-0.017$^a$</td>
<td>-0.001$^a$</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>$\phi-\theta$ (m$^3$ m$^{-3}$)</td>
<td>-0.785</td>
<td>-6.365</td>
<td>-0.951</td>
<td>-4.805</td>
<td>-0.889</td>
<td></td>
</tr>
<tr>
<td>$Q$ (m$^3$ m$^{-3}$)</td>
<td>0.749</td>
<td>2.999</td>
<td>2.406</td>
<td>1.069</td>
<td>0.533</td>
<td></td>
</tr>
</tbody>
</table>
| B (a $\leftrightarrow PR$, $b \leftrightarrow \theta$; $c \leftrightarrow intercept$; $\theta$—water content; $\phi$—porosity (or $\phi = \theta_s$—saturated water content); $Q$—content of sand; $PR$—penetration resistance; $H_c$—hardness index; $\leftrightarrow$ means parameter corresponding to a given variable. $^a$Not statistically significant. The data for the silt loam (B) are from Usowicz et al., 2006a. Beta coefficients, being the regression coefficients, were derived from standardization of variables to a mean of zero and a standard deviation of one to compare the relative contribution of each independent variable in the prediction of the thermal conductivity. B coefficients that are not standardized were used to calculate the thermal conductivity from the equations derived with the procedure of Fixed Non-linear Regression (StatSoft, Inc., 2004).

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Sandy loam</th>
<th>Silt loam (A)</th>
<th>Silt loam (B)</th>
<th>Sandy loam + silt loam (A)</th>
<th>Sandy loam + silt loam (A) + silt loam (B)</th>
<th>Snow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (1)</td>
<td>Eq. (2)</td>
<td>Eq. (1)</td>
<td>Eq. (2)</td>
<td>Eq. (1)</td>
<td>Eq. (2)</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.138</td>
<td>0.102</td>
<td>0.077</td>
<td>0.077</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>MRE</td>
<td>29.3</td>
<td>23.0</td>
<td>23.4</td>
<td>24.5</td>
<td>34.2</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.968</td>
<td>0.968</td>
<td>0.923</td>
<td>0.958</td>
<td>0.942</td>
<td></td>
</tr>
</tbody>
</table>

RMSE (W m$^{-1}$ K$^{-1}$)—root mean square error, MRE (%)—maximum relative error and $R^2$—determination coefficients.
sandy loam and silt loam (A) soils with greater content of sand compared to the silt loam (B) soil are a reflection of greater means of penetration resistance and bulk density in the sandy loam soil \((2680 \text{ kPa and } 1.609 \times 10^3 \text{ kg m}^{-3})\) and in the silt loam (A) soil \((2467 \text{ kPa and } 1.53 \times 10^3 \text{ kg m}^{-3})\) than the silt loam (B) soil \((2400 \text{ kPa and } 1.25 \times 10^3 \text{ kg m}^{-3})\), and the opposite order was found for water content being 0.142, 0.222 and 0.275 m\(^3\) m\(^{-3}\) in the sandy loam, silt loam (A) and silt loam (B) soils, respectively. In general, the values of beta for the content of sand increased successively in soils with increasing content of sand. Moreover, the negative values of beta for penetration resistance for the two soils with a greater content of sand (sandy loam and silt loam (A)) can be likely due to insufficient number of data for given levels of bulk density and water content although total number of the data was 300 that seems quite reasonable.

When the combined data for sandy loam and silt loam (A) soils were analysed, the value of beta for porosity was lower for the equation without than for that with sand, however the inverse was true after adding data of the silt loam (B) soil. As to penetration resistance, the values of beta were negative for combined data of the sandy loam and silt loam (A) soils, and positive in the case of data from all three soils. A substantial effect of the silt loam (B) soil on the beta values is likely due to considerably lower content of sand in this soil than in the other two soils, as indicated in the preceding paragraph. The values of beta for the content of sand for both groups of soils were considerably greater than for each individual soil (Table 2).

We focused in this study on the effects of sand content rather than silt and clay fractions due to a substantially greater thermal conductivity (approximately three times) of the former. However, the results obtained imply that in less sandy soils replacing sand by dominating finer fraction or its adding as an independent variable could improve the performance of the regression equations. To confirm this, further studies including porous media in a wide range of grain compositions are needed.

As to snow, the values of beta indicate that relative contributions of penetration resistance, as characterized by the index of hardness and by porosity, in the prediction of the thermal conductivity were similar, but oppositely signed (Table 2). The opposite signs indicate that increase of porosity and index of hardness leads to decrease and increase of the thermal conductivity, respectively.

Comparison of the data for soil and snow in Table 2 indicates that the values of beta with respect to penetration resistance were considerably greater for snow than for soils, however the inverse was true with respect to porosity. The differentiated relative contribution of particular independent variables in the prediction of thermal conductivity of the porous media can be due to that in the case of soil at given bulk density the significant factor influencing penetration resistance is water content and in the case of snow—density. Moreover, porosity of soil at a given density, in contrast to that of snow, decreases with increasing water content.

B coefficients, associated with beta coefficients, were used in the regression equations to predict the thermal conductivities (Table 2). The coefficients represent regression coefficients \(a, b, b_1, c\) in a given equation; \(a\) is associated with root square of \(PR\) or index of hardness \(H_s\), \(b\)—with porosity of soil \(\phi—\theta\) or porosity of snow \(\phi; b_1—\) with sand content \(Q\) and \(c\) is the intercept.

The measured and predicted thermal conductivity by regression equations (with 95% confidence) based on penetration resistance and porosity data for individual and combined soils with and without sand and for snow, as well as distribution of residuals with normal theoretical distribution and statistical parameters describing the performance of the equations, are presented in Figs. 1 and 2 and in Table 3. The highest range and maximum values of the thermal conductivity \((0.4–2.9 \text{ W m}^{-1} \text{ K}^{-1})\) occurred in the sandy loam soil (Fig. 1a). As expected the maximum values were somewhat lower in the silt loam (A) soil \((2.4 \text{ W m}^{-1} \text{ K}^{-1})\) (Fig. 1b) and considerably lower in the silt loam (B) soil \((1.6 \text{ W m}^{-1} \text{ K}^{-1})\) (Usowicz et al., 2006a). In the case of snow, the maximum value \((0.59 \text{ W m}^{-1} \text{ K}^{-1})\) was lower up to several times (Fig. 1c). However, dispersions for data for individual and combined soils were lower with than without consideration of sand.

As indicated by statistical parameters (RMSE, MRE, \(R^2\)), the performance of the regression equations with sand is better with than without adding the content of sand, except for MRE with respect to the silt loam (A) soil where the effect was statistically insignificant (Table 3). Comparison of the observed and predicted thermal conductivity for the three individual soils without consideration of sand indicates a considerable dispersion of the residuals varying from –0.45 to 0.3 with mean of near zero. The data in Table 3 indicate that the penetration resistance and porosity accounted for 92–97% of the variation in thermal conductivity, with RMSE varying from 0.067 to \(0.138 \text{ W m}^{-1} \text{ K}^{-1}\) and MRE from 23.4% to 34.2% depending on soil type. A good agreement of the distribution of residuals with the theoretical normal distribution (solid line) with consideration of content of sand, as shown in Fig. 1, implies that the dispersion of the residuals has a random nature and thereby lack of structural component that could considerably deform distribution of predicted values.

Adding the content of the sand as an independent variable to penetration resistance and porosity in the regression Eq. (2) resulted in improvement of the thermal conductivity predictions as compared with using just soil penetration resistance and porosity (Eq. (1)) (Table 3). The ranges of the \(R^2\), RMSE and MRE of the equations with sand for all individual soils were 0.958–0.97, 0.055–0.102 \text{ W m}^{-1} \text{ K}^{-1}\) and 23.0–27.3%, respectively. Also in the case of data for combined soils (two or three soils) the predictability of the regression equations with sand \((R^2 = 0.933–0.947)\) was substantially better than without
Fig. 1. Observed thermal conductivity and predicted by regression equation based on porosity, penetration resistance and sand content in unit volume of soil for soils and based on porosity and hardness for snow with 95% confidence and histogram of residuals with the theoretical normal distribution (solid line); (a) sandy loam, (b) silt loam (A) and (c) snow, with the numbers of pairs of data at 160, 140 and 178, respectively.
Fig. 2. Observed thermal conductivity and predicted by regression equation based on porosity and penetration resistance with 95% confidence and histogram of residuals with the theoretical normal distribution (solid line) with (b and d) and without (a and c) consideration of sand; (a and b) sandy loam and silt loam (A) together; (c and d) sandy loam, silt loam (A) and silt loam (B). The numbers of pairs of data are 300 for sandy loam and silt loam (A) and 1740 for sandy loam, silt loam (A) and silt loam (B).
sand ($R^2 = 0.399–0.591$) (Table 3). Comparison of histogram of residuals with theoretical normal distribution (solid line) (Fig. 2a and c) indicates a lack of agreement between the residuals and values of the theoretical normal distribution for the regression equations based on penetration resistance and porosity for the data from combined soils (two and three soils). This implies that the dispersion of the residuals has no random nature and thereby indicates the occurrence of structural component that can considerably deform the distribution of predicted values. After adding the content of sand to the regression, the agreement between the residuals and the theoretical normal distribution was better for the data set of the sandy loam and silt loam (A) soils than for all three soils (Fig. 2b and d).

The courses of the thermal conductivity with clearly separated concentrations of points in Fig. 2a and c can be referred to the particular soils used in this study. From the comparison of the above figures with Fig. 1, it results that the upper concentration with the greatest values of thermal conductivity refers to the sandy loam soil (highest in sand), the middle one—to the silt loam (A) soil, and the lower one—to the silt loam (B) soil (lowest in sand content) (Usowicz et al., 2006a). Weak agreement between the observed and predicted thermal conductivity using the equation for combined soils (Fig. 2 and Table 3), and good agreement for the individual soils (Fig. 1 and Table 3), indicate that satisfactory thermal conductivity predictions based only on porosity and penetration resistance can be obtained with respect to soils with not much different content of sand.

Our results indicate that the performance of the regression equation based on porosity and penetration resistance with consideration of the sand using data of two soils (sandy loam and silt loam (A)) with the content of sand fraction different by 20% was better (Fig. 2b) than after adding data of the third soil (silt loam (B)) with content of sand lower by 55% (Fig. 2d). This evaluation is based on MRE that was more than two times greater for the data from three than from two soils. However, values of the remaining statistical parameters (RMSE, $R^2$) were similar and even slightly better for the data from three soils. This can be associated with more than four times greater number of data for the silt loam (B) soil than for the sandy loam and silt loam (A) soils together. It is worthy of noting that the consideration of content of sand in the regression equation results in better thermal conductivity predictions of the soils with relatively not much different content of sand. However, the thermal conductivity predictions can be worse for combined soils with great differences in sand content, in particular at high porosities (the points below the dotted line) (Fig. 2d).

In the case of snow, the predictive capability of thermal conductivity with the regression equations based on index of hardness and porosity (Eq. (3)) was better than with the regression based on just density (Eq. (4)). Corresponding $R^2$, RMSE and MRE values of the respective equations were 0.657, 0.076 W m$^{-1}$ K$^{-1}$, 348% and 0.501, 0.091 W m$^{-1}$ K$^{-1}$, 402% (Table 3). Taking into consideration that MRE values were relatively high and occurred sporadically, we calculated also average MRE that was several times lower than MRE and thus confirmed better performance of the suggested Eq. (3) (45.6%) than Eq. (4) (48.4%). A good agreement of the residuals with the theoretical normal distribution (Fig. 1c, solid line) with respect to snow, similarly as for all individual soils, was obtained. It is worth of noting that the index of hardness, considerably increasing the accuracy of predicting the snow thermal conductivity, can be obtained from simple measurement of penetration resistance by fist, several fingers, a pencil or knife, in accordance with the International Classification of Snow on the Ground (Colbeck et al., 1992 quoted by Sturm et al., 2002). One can expect that the use of more precise methods of measuring penetration resistance can further improve the predictive capability of the snow thermal conductivity using the suggested regression equation.

The results in Fig. 1, referring to the equations with three independent variables (penetration resistance, porosity and sand content), indicate that the thermal conductivity of the sandy loam and silt loam (A) soils with greater content of sand compared to the silt loam (B) are characterised by a greater dispersion of the thermal conductivity. For example, the thermal conductivity of the sandy loam soil compared to the silt loam (B) soil is characterised by a greater dispersion, by approximately two times, and a greater range of the thermal conductivity by 1.2 W m$^{-1}$ K$^{-1}$ (difference between the maximum thermal conductivity of sandy loam soil, 2.8 W m$^{-1}$ K$^{-1}$, and silt loam (B) soil, 1.6 W m$^{-1}$ K$^{-1}$).

In the case of snow, the values of dispersion of the residuals varied from 0.2 to 0.25 W m$^{-1}$ K$^{-1}$ and the range of the thermal conductivity was relatively narrow (0.032–0.587 W m$^{-1}$ K$^{-1}$). The considerable range of the dispersion values of the snow thermal conductivity compared with its narrow range of changes indicate that prediction of thermal conductivity based on easily measured independent variables with respect to snow is not as satisfactory as it is with the mineral soils (Fig. 1). The results indicate the more complex nature of snow, understanding of which needs further research. The values of dispersion of the thermal conductivity of soil and snow were similar, whereas its range was considerably lower for snow than for the soils. It is worthy to note that the predictability of the thermal conductivity of the porous media based on porosity and penetration resistance is more accurate than that based on water content for soil (Usowicz et al., 2006a, b) and on density for snow (Sturm et al., 2002).

The experimental and theoretical studies aiming at relating the mechanical and thermal properties of porous media have been recently enhanced (Marczewski et al., 2004; Usowicz et al., 2006b). They are supported by the fact that the weathering of rock and resulting breaking up
of regolith can fill in the porous space formed by cracking (e.g. in meteorites) or form a loose surface layer (Strait and Consolmagno, 2005). Filling in the cracks by weathered material can increase the number of contacts and the contact area between the grains and, consequently, a greater thermal conductivity in the area of the cracks. The regolith loose surface layer will have a greater porosity and lower cohesion and associated penetration resistance and number of contacts between the grains, resulting in a lower thermal conductivity than non-weathered rock. Taking into consideration certain limitations of the thermal conductivity measurements associated with low thermal conductivity of the porous media (Seiferlin et al., 2006), the use of indirect method for predicting the thermal conductivity based on easily measured penetration resistance, texture and porosity can be one alternative approach that can help minimise the uncertainties associated with measurements.

The terrestrial porous media used in this study, as characterised by different content of sand, water and density (porosity), and snow of different density and hardness, can be certain analogues of extraterrestrial objects, e.g. granulated ice in some comets or other extraterrestrial objects consisting of minerals, ice and their mixtures (Gori and Corasaniti, 2004).

Measurements of penetration resistance were carried out in the past and are intended in future extraterrestrial missions. This quantity characterizes not only the mechanical properties of porous media, but also can be used in the evaluation of grain size distribution of extraterrestrial surface (Kargl and Zöhrer, 2006) and aggregate size distribution of terrestrial soils (Pawlowski et al., 1996) and is indicative of the size of contacts between the grains which, in turn, can be used to estimate thermal conductivity. Increase of the size of contacts results in a greater thermal conductivity and vice versa. Measurements of penetration resistance and thermal conductivity are intended in the mission Rosetta (Marczewski et al., 2004; Spohn et al., 2007). Such measurements, together with simulations of thermal conductivity under laboratory conditions of extraterrestrial objects (Seiferlin et al., 1996) and terrestrial soils (Usowicz et al., 2006a), can be useful not only for the estimation of thermal conductivity, and thus help minimise the uncertainties associated with different methods used under space conditions, but also for the estimation of the size of grains and contacts in extraterrestrial media. In addition, the regression equations proposed in this study can be of some help in estimating porosity of extraterrestrial porous media based on the future measurement data of the thermal conductivity and penetration resistance from the Rosetta mission.

4. Conclusions

We have presented the regression equations for prediction of thermal conductivity of some terrestrial soils and snow based on easily measured or available quantities. The predictive capability of the regression equations depends on the number of independent variables and was better for the equations based on penetration resistance, porosity and content of sand than for those based on penetration resistance and porosity. The equations with consideration of the sand content allow predicting thermal conductivity satisfactorily for soils with not much different content of sand (<20%). However, in the case of soils with much different content of sand separate regression equations should be developed for each soil. The same type of regression equation can be used for predicting the thermal conductivity of soils with differentiated content of sand but with less accuracy than using separate regression equations for particular soils. The regression equations for predicting the snow thermal conductivity based on the index of hardness and porosity are characterised by better predictive capability than those based on density alone. The independent variables used in the regression equation for prediction of thermal conductivity are also indicative of the mechanical behaviour of the porous media.

Acknowledgement

This work was funded in part by the Polish Ministry of Science and Higher Education (Grant no. N305 046 31/1707).

References


